

MATH 134A Review: System of Linear Equations

Facts to Know

A system of linear equations with two equations and two variables is

$$\begin{cases} ax + by = u \\ cx + dy = v \end{cases}$$

Here x, y are variables, and a, b, c, d, u, v are constants

The corresponding matrix equation is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$"A\vec{x} = \vec{b}"$$

A solution is a vector, and in particular for this setting a solution

is of the form $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = (x_0, y_0)$ and $ax_0 + by_0 = u$
and $cx_0 + dy_0 = v$

A method to find a solution is

Special case $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible i.e. $ad - bc \neq 0$

(1) Find $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$ (2) Multiply $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}$ on the left of both sides $A\vec{x} = \vec{b}$

Recall the inverse of a 2×2 matrix is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples

1. A restaurant manager wants to purchase 200 sets of dishes. One design costs \$25 per set, while another costs \$45 per set. If she only has \$7400 to spend, how many of each design should be ordered?

dishes x y

$$\begin{cases} x + y = 200 \\ 25x + 45y = 7400 \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ 25 & 45 \end{bmatrix}^{-1} = \frac{1}{45 - 25} \begin{bmatrix} 45 & -1 \\ -25 & 1 \end{bmatrix}$$
$$= \frac{1}{20} \cdot \begin{bmatrix} 45 & -1 \\ -25 & 1 \end{bmatrix}$$

$A\vec{x} = \vec{b}$

$$\frac{1}{20} \begin{bmatrix} 45 & -1 \\ -25 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 25 & 45 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 45 & -1 \\ -25 & 1 \end{bmatrix} \begin{bmatrix} 200 \\ 7400 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{20} \begin{bmatrix} (9000 - 7400) \\ (-5000 + 7400) \end{bmatrix}$$
$$= \frac{1}{20} \begin{bmatrix} 1600 \\ 2400 \end{bmatrix} = \begin{bmatrix} 80 \\ 120 \end{bmatrix}$$

80 dishes bought which costs \$ 25

120 dishes bought which costs \$ 45

2. A movie theater charges \$9.00 for adults and \$7.00 for senior citizens. On a day when 325 people paid an admission, the total receipts were \$2495. How many who paid were adults? How many were seniors?

$$x = \# \text{ Adults}$$

$$y = \# \text{ S.C.}$$

$$\begin{cases} x + y = 325 \\ 9x + 7y = 2495 \end{cases}$$

$$\textcircled{1} \begin{bmatrix} 1 & 1 \\ 9 & 7 \end{bmatrix}^{-1} = \frac{1}{7-9} \begin{bmatrix} 7 & -1 \\ -9 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 7 & -1 \\ -9 & 1 \end{bmatrix} = A^{-1}$$

$$A\vec{x} = \vec{b}$$

$$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 7 & -1 \\ -9 & 1 \end{bmatrix} \begin{bmatrix} 325 \\ 2495 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} (2275 - 2495) \\ (-2925 + 2495) \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} -220 \\ -430 \end{bmatrix}$$

$$= \begin{bmatrix} 110 \\ 215 \end{bmatrix}$$

110 adults and 215 senior citizens

